FÍSICA da MATÉRIA CONDENSADA

Mestrado em Engenharia Física Tecnológica

Série 4b

1. In the usual treatment of the BCS equations, one usually solves the equation

$$E_k^2 = (\epsilon_k - \mu)^2 + |\Delta_k|^2$$

for $E_k = +|\sqrt{(\epsilon_k - \mu)^2 + |\Delta_k|^2}|$. However, there is also another solution for $E_k = -|\sqrt{(\epsilon_k - \mu)^2 + |\Delta_k|^2}|$.

Study this solution and interpret it physically, checking what happens, if we change the sign of E_k, to the usual BCS equations:

$$|\psi_0\rangle = \prod_k (u_k + v_k c_k^\dagger c_{-k}^\dagger)|0\rangle$$

$$\begin{aligned} \gamma_k &= u_k c_k - v_k c_{-k}^\dagger \\ \gamma_{-k} &= u_k c_{-k} + v_k c_k^\dagger \end{aligned}$$

$$\begin{aligned} \gamma_k|\psi_0\rangle &= 0 \\ \gamma_{-k}|\psi_0\rangle &= 0 \end{aligned}$$

$$\Delta_k = 2E_k u_k^* v_k \qquad E_k = \sqrt{(\epsilon_k - \mu)^2 + |\Delta_k|^2}$$

$$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\epsilon_k - \mu}{E_k}\right) \qquad |v_k|^2 = \frac{1}{2}\left(1 - \frac{\epsilon_k - \mu}{E_k}\right)$$

$$\begin{aligned} \langle c_k^\dagger c_k\rangle &= |u_k|^2 f(E_k) + |v_k|^2(1 - f(E_k)) \quad T \geq 0^\circ \\ &= |v_k|^2 = \langle\psi_0|c_k^\dagger c_k|\psi_0\rangle \quad T = 0^\circ \end{aligned}$$

$$\Delta_k = -\sum_{k'} V_{kk'} < c_{-k'} c_{k'} >= -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \tanh(\frac{\beta E_{k'}}{2})$$

$$f(E_k) = \frac{1}{e^{\beta E_k} + 1}$$

where the simplified notation $k = \{\vec{k} \uparrow\}$ and $-k = \{-\vec{k} \downarrow\}$ was used.